Section A

1. If A is a square matrix of order n then |adj A| is

a) $|A|^2$ b) $|A|^n$ c) $|A|^{n-1}$ d) |A|

2. If $\vec{p}, \vec{q}$ and $\vec{p}+\vec{q}$ are vectors of magnitude λ then the magnitude of $\vec{p}-\vec{q}$ is

a) 2λ b) $\sqrt{3}\lambda$ c) $\sqrt{2}\lambda$ d) 1

3. The value of $i + i^{22} + i^{23} + i^{24} + i^{25}$ is

a) I b) –i c) 1 d) -1

4. $\lim_{x\to\infty} \frac{x^2}{e^x}$ =

a) 2 b) 0 c) ∞ d) 1

5. The point of intersection of the tangents at $t_1 = t$ and $t_2 = 3t$ to the parabola $y^2 = 8x$ is

a) $(6t^2,8t)$ b) $(8t,6t^2)$ c) $(t^2,4t)$ d) $(4t,t^2)$

6. In which region the curve $y^2(a+x) = x^2(3a-x)$ does not lie

a) $x>0$ b) $0<x<3a$

c) $z \leq -a$ and $x > 3a$ d) $-a < x < 3a$

7. The value of $\int_0^{\pi/4} \cos^3 2x\, dx$ is

a) 2/3 b) 1/3 c) 0 d) 2π/3

8.The differential equation obtained by eliminating 'a' and 'b' from $y = a e^{3x} + be^{-3x}$ is

a) y"+ 9y=0 b) y"- 9y=0

c) y" - 9y′ = 0 d) y"+ 9x =0

9. Which of the following is contradiction?

a) p∨q b) p∧q c) p∨(~p) d) p∧(~p)

10. If f(x) is the p.d.f of a normal distribution with mean μ then $\int_{-\infty}^{\infty} f(x)dx$ is

a) 1 b) 0.5 c) 0 d) 0.25

11. The angle between the vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}\times\vec{b}| = \vec{a}.\vec{b}$ is

a) π/4 b) π/3 c) π/6 d) π/2

12. The equation of the tangent at (-3,1) to the parabola $x^2 = 9y$ is

a) $3x - 2y - 3 = 0$ b) $2x - 3y + 3 = 0$

c) $2x + 3y + 3 = 0$ d) $3x + 2y + 3 = 0$

13. The differential of xtanx is

a) $(x \sec^2 x + \tan^2 x)\, dx$

b) $(x \sec^2 x - \tan x)\, dx$

c) $x \sec^2 x\, dx$

d) $(x \sec^2 x + \tan x)dx$

14. The order and degree of the differential equation $y' + (y'')^2 = (x + y'')^2$ are

a) 2,2 b) 2,1 c) 1,2 d) 1,1

15. If x is a continous random variable then $p(xy \geq a) =$

a) $p(x < a)$ b) $1 - p(x > a)$

c) $p(x > a)$ d) $1 - p(x \leq a - 1)$

16. A system contains only one equation in the variables x,y then the system has

a) one solution b) two solutions

c) no solutions d)many solutions

17. If $z = r\, e^{i\theta}$ then the value of $|e^{iz}|$ is

a) $e^{-r\sin\theta}$ b) $re^{-r\sin\theta}$ c) $e^{-r\cos\theta}$ d) $re^{-r\cos\theta}$

18. The area between the curves $y = e^x$ and $y = e^{-x}$
From x = 0 to x = 2 is

a) $e^2 + \frac{1}{e^2}$ b) $(e - \frac{1^2}{e})$ c) $e + \frac{1}{e} + 1$ d) $\left(e + \frac{1}{e}\right)^2$

19. Angle between $x^2 - y^2 = 16$ and $xy = 4$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) 0 d) $\tan^{-1}\left(\frac{1}{4}\right)$

20. In the group G = { [2],[4],[6],[8]} under multiplication modulo 10. The inverse of the element [4] is _____

a) [2] b) [4] c)[6] d)[8]

Section B

(i) Answer any SEVEN questions

(ii) Question No.30 is compulsory and choose any SIX questions from the remaining

21. Find the rank of the matrix $\begin{bmatrix} a & b & c \\ a-b & b-c & c-a \\ a+b & b+c & c+a \end{bmatrix}$

22. Find the centre of the sphere

$(\bar{r} - \bar{a})$ o $(\bar{r} - \bar{b})$ = 0

23. Find the cube roots of -1

24. Find the equation of the parabola having vertex (1,2) and latus rectum x = 3.

25. Find the expansion of e^{-x}.

26. Find the local maximum and local minimum values of the function t + cost

27. Evaluate $\int_0^a \sqrt{a^2 - x^2}\,dx$

28. Find the particular integral of $D^2y = 7$

29. For any group G, the identity element is the only element of order 1

30. If the mean of a poisson random variable X is λ and Var $(\lambda x + \lambda^2 + 2\lambda) = 8$. Find the mean

Section C

(i) Answer any SEVEN questions

(ii) Question number 40 is compulsory and choose any SIX questions from the remaining

31. A = $\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ and $XB^{-1} = A^{-1}$ Find X.

32. Find the equation of the line through the point (1,-2,3) and perpendicular to the plane x + 3y – z = 3. Hence find the foot of the perpendicular from the point on the plane.

33. Simplify $\left[\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^8$

34. Find the equation of the two tangents that can be drawn from the point (2,-3) to the parabola

$y^2 = 4x$.

35. If $u = x^3 + y^3 + z^3 - 3xyz$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

36. Verify Lagranges' mean value theorem for the function $f(x) = x^3 – 5x^2 – 3x$ is [1,3].

37. In a bionomial distribution n = 32, variance = 6. Find the mean

38. Find the volume of a sphere of radius 'a' units

39. State and prove cancellation laws of a group.

40. Solve tanx $\frac{dy}{dx} + sec^2x.y = tan^3x$

Section D

Answer all the questions

41. A bag contains three types of coins Re.1, Re.2 and Re. 5. There are 30 coins amounting to Rs. 100 in total. Find the number of coins in each category.

Or

Solve $x^9 + x^5 - x^4 - 1 = 0$

42. Prove that attitudes of a triangle are concurrent by vector method

Or

Show that the volume of the largest right circular cone that can be inscribed is a sphere of radius 'a' is $\frac{8}{27}$ times volume of the sphere.

43. Find the vector and Cartesian equation of the plane containing the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$ and perpendicular the plane 3x + 2y + z = 0

Or

Find the length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

44. An arch is in the form of a semi ellipse whose span is 48 feet wide. The height of the arch is 20 feet. How wide is the arch at a height of 10 feet above the base

Or

If X is normally distributed with mean 6 and standard deviation 5. Find (i) p ($0 \leq X \leq 8$) (ii) p (|X-6|<10).

45. If the curve $y^2 = 4ax$ and $xy = c^2$ are orthogonal then prove that $c^4 = 32\ a^4$

Or

A cup of coffee 100°C is placed in a room whose temperature is 15°C and it cools to 60 °C in 5 minutes. Find its temperature after a further interval of 5 minutes.

46. Find the equation of the rectangular hyperbola which has for one of its asymptotes the line x + 2y – 5 = 0 and passes through the points (6,0) and (-3,0).

Or

Verify Euler's theorem for the function $\frac{1}{\sqrt{x^2+y^2}}$.

47. Find the area bounded by x – axis and an arch of the cycloid x = a(2t – sin2t) y = a(1-cos2t)

Or

Prove that (Z, - {[0]},7) is an abelian group